## Design and Analysis of Algorithms 1 Semester, 2024-25, Indian Statistical Institute, Bangalore

## Mid term Examination ; marks 60, Time Limit - 2 hours

September 9, 2024

## 1. Answer briefly

(a) Formally, what do we mean by the notation f(n) = O(g(n))? How is it different from saying that  $f(n) = \Theta(g(n))$ ?

[3x5=15]

[4]

- (b) What is the order in which you print the nodes in binary search tree so as to create a sorted order from largest to smallest? sketch the recursive pseudo-code.
- (c) When using DFS to find articulation points of an undirected graph G, we compute a lowest value for each vertex. Conceptually what does this value indicate? Show the way to compute this value.
- (d) Consider the recurrence relation below representing the complexity of a certain algorithm:

$$T(1) = 1;$$
  $T(n) = 2T(n/2) + n$   $n > 1$ 

Derive the complexity by drawing the recursion tree and summing the series.

(e) Use the master theorem (illustrating the idea of the theorem) to find the complexity of an algorithm expressed by this recursive definition:

$$T(1) = 1;$$
  $T(n) = 3T(n/2) + n$   $n > 1$ 

- (a) Write out Prim's algorithm which grows a min weight spanning tree from a given source node s assuming an adjaceny list representation for the graph. Mention any data structures used for an efficient implementation. Derive the complexity with that implementation. [10]
  - (b) Which step(s) takes different time if the graph was represented as an adjacency matrix instead of an adjacency list. Compute the complexity of the algorithm with the adjacency matrix representation. [5]
- 3. You are given an array which has *n* distinct values. The values are arranged so that they monotonically increase up to a maximum and then after that point monotonically decrease. Eg. 1 4 9 10 12 8 2 0 is such a sequence. Of course, the purely increasing or purely decreasing part could be empty. Give an efficient algorithm (better than linear time) to find the maximum value in the array and analyse its complexity. [15]
- 4. (a) Sketch Dijkstra's algorithm.
  - (b) Show its progress on the following graph. Clearly indicate the initial d values and the d values after each time a vertex is picked. Also clearly indicate the picked vertex at each step. Also clearly mention or graphically show show the

edges used in the shortest paths to each vertex.



(c) In Dijkstra's algorithm, at what point does it make a mistake if we have negative edge weights (even without negative cycles). Why? [3]